Matrices

Multiple Choice Questions

Q: 1 $M = \begin{bmatrix} 2 & 3 \\ 6 & 4 \end{bmatrix}$

Given below are two statements in relation with the above matrix- one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion *(A)* : The matrix M cannot be expressed as the sum of a symmetric and a skew-symmetric matrix.

Reason *(R)* : The matrix M is neither a symmetric matrix nor a skew-symmetric matrix.

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is false but (R) is true.
4. Both (A) and (R) are false.

Free Response Questions

Q: 2 **The following are two non-zero matrices M and N:** **[1]**

$$M = \begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix} \quad N = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

What is the necessary condition for their product to be a zero matrix? Show your work and give valid reason.

Q: 3 **Shown below is matrix P.** **[3]**

$$P = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

Find one such matrix Q such that (P + Q) is a skew-symmetric matrix. Show your work.

Q: 4 **Drona is solving a bank robbery. He has a note from one of his informants, Arjun, which contains the robber's name in the form of an encoded matrix N.** **[5]**

♦ The decoded name is stored in the form of a matrix M with numbers representing letters, such that 1 = A, 2 = B, 3 = C, and so on until 26 = Z.
♦ The entries are to be read column-wise.
♦ Arjun's notes to Drona are encoded using an encoder matrix, E.
♦ Matrix E is multiplied with Matrix M, to get the matrix in the note, N.

Matrices N and E are given below.

$$\begin{bmatrix} 62 & 52 \\ 25 & 21 \\ -51 & -51 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -1 & -2 & -2 \end{bmatrix}$$

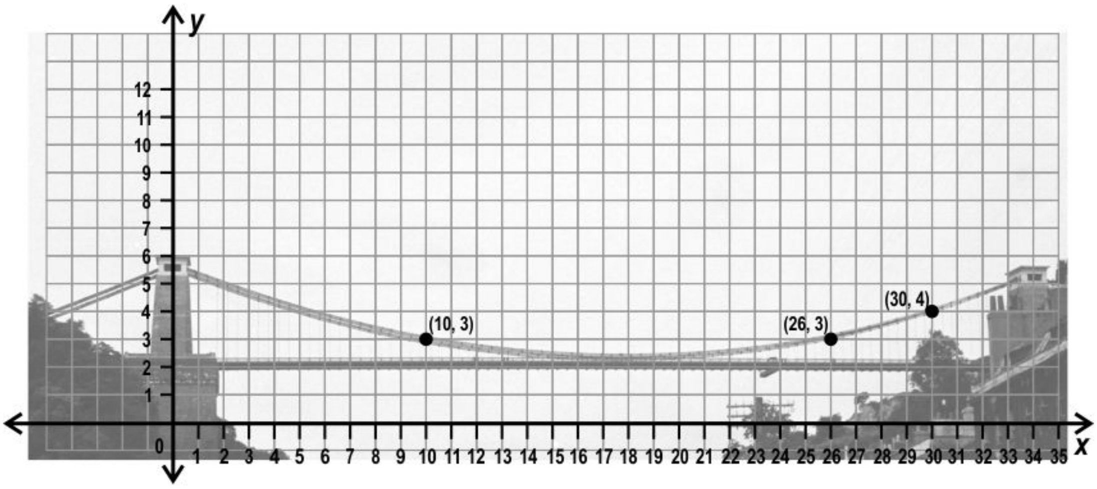
Matrix the note (N)	Encoder Matrix (E)

Find the key to decode the matrix N, and use it to find the robber's name. Show your work.

(Note: Encoding a matrix means to convert it to a coded form; Decoding a matrix means to convert it from coded form to normal form.)

Q: 5 As part of an architecture project, Henry needs to create a miniature model of the Clifton Bridge, located in Bristol, UK. In order to do this, he must first find the equation of a parabola equivalent to the one made in Clifton Bridge. **[5]**

He found a picture of the Clifton bridge, and placed it on a coordinate grid, as shown below. He noted that the parabola crossed the points (10, 3), (26, 3) and (30, 4) on the grid.



If the required parabola is of the form $y = ax^2 + bx + c$, then help Henry by finding a, b and c, and hence finding the equation of the parabola. Show your work.

[redacted]

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3

<table>
<tr><th>Q.No</th><th>What to look for</th><th>Marks</th></tr>
<tr><td rowspan="2">2</td><td>Writes the product of the matrices M and N as follows:

$$MN = \begin{bmatrix} ar & as \\ br & bs \end{bmatrix}$$</td><td>0.5</td></tr>
<tr><td>Writes that the necessary condition for MN to be a zero matrix is that the values of r and s must always be 0 because a and b cannot be zero as M is a non-zero matrix.</td><td>0.5</td></tr>
<tr><td rowspan="3">3</td><td>Represents the matrix (P + Q) as shown below:

$$P + Q = \begin{bmatrix} a+3 & b-1 \\ c+1 & d-2 \end{bmatrix}$$
$$\text{where } Q = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$</td><td>0.5</td></tr>
<tr><td>Uses the skew-symmetric relation (P + Q) = -(P + Q)' to write the equation as:

$$\begin{bmatrix} a+3 & b-1 \\ c+1 & d-2 \end{bmatrix} = \begin{bmatrix} -(a+3) & -(c+1) \\ 1-b & 2-d \end{bmatrix}$$</td><td>1</td></tr>
<tr><td>Finds the value of a as (-3), d as 2 and the relation b = - c using the equality of matrices in the above step and solve the following equations:

a + 3 = -(a + 3)

d - 2 = 2 - d

and

b - 1 = -(c + 1)</td><td>1</td></tr>
</table>

Q.No	What to look for	Marks
	Finds the matrix Q as shown below where the numerical values of *b* and *c* are such that *c* = - *b* : $$Q = \begin{bmatrix} -3 & b \\ -b & 2 \end{bmatrix}$$	**0.5**
4	**Notes that the determinant of Matrix E ≠ 0.** **Writes that, since EM = N, we can multiply both sides by E^{-1} to get E^{-1} EM = E^{-1} N.** **Simplifies the above expression to get M = E^{-1} N.**	**1**

Q.No	What to look for	Marks
	Finds the inverse of matrix E using elementary row operations. **The solution may look as follows:** $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -1 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} E$ $R_3 \to R_3 + R_1$ $\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} E$ $R_1 \to R_1 - 2R_2 - R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} E$ $R_2 \to R_2 - R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} E$ $\Rightarrow E^{-1} = \begin{bmatrix} 0 & -2 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ **(Award full marks if inverse is found using adj(E) ÷ det(E).)**	2

Q.No	What to look for	Marks
	Multiplies E $^{-1}$ with N as follows to get M: $\begin{bmatrix} 0 & -2 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 62 & 52 \\ 25 & 21 \\ -51 & -51 \end{bmatrix}$ $= \begin{bmatrix} 0+(-50)+51 & 0+(-42)+51 \\ (-62)+25+51 & (-52)+21+51 \\ 62+0+(-51) & 52+0+(-51) \end{bmatrix}$ $= \begin{bmatrix} 1 & 9 \\ 14 & 20 \\ 11 & 1 \end{bmatrix}$	1.5
	Decodes the letters representing the numbers and reads them column-wise to get the robber's name as ANKITA.	0.5
5	**Writes the system of equations as:** **100 a + 10 b + c = 3** **676 a + 26 b + c = 3** **900 a + 30 b + c = 4**	0.5
	Writes the above system of equations in the form AX = B as follows: $\begin{pmatrix} 100 & 10 & 1 \\ 676 & 26 & 1 \\ 900 & 30 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$	0.5
	Finds |A| as 1(20280 - 23400) - 1(3000 - 9000) + 1(2600 - 6760) = -1280. **Writes that A $^{-1}$ exists as |A| ≠ 0.**	0.5
	Finds A $^{-1}$ as: $A^{-1} = \begin{pmatrix} \frac{1}{320} & -\frac{1}{64} & \frac{1}{80} \\ -\frac{7}{40} & \frac{5}{8} & -\frac{9}{20} \\ \frac{39}{16} & -\frac{75}{16} & \frac{13}{4} \end{pmatrix}$ **(Award 1 mark if only all the cofactors are found correctly.)**	2

Q.No	What to look for	Marks
	Finds the values of *a*, *b* and *c* as $\frac{1}{80}$, - $\frac{9}{20}$ and $\frac{25}{4}$ respectively by solving for matrix X = A $^{-1}$ B as follows: $$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{1}{320} & -\frac{1}{64} & \frac{1}{80} \\ -\frac{7}{40} & \frac{5}{8} & -\frac{9}{20} \\ \frac{39}{16} & -\frac{75}{16} & \frac{13}{4} \end{pmatrix} \times \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{80} \\ -\frac{9}{20} \\ \frac{25}{4} \end{pmatrix}$$	1
	Writes that the equation of a parabola equivalent to that of the Clifton bridge is $\frac{1}{80}x^2 - \frac{9}{20}x + \frac{25}{4}$.	0.5